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Quantum Theoretical Physics is Statistical and Relativistic

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We present a new theoretical framework for the "quantum" mechanism. We base it on a strict deterministic behavior of single systems. The conventional QM equation, however, is found to describe statistical results of many classical systems. We will see, moreover, that a rigorous synthesis of our theory requires relativistic kinematics. *So, QM is not only a classical statistical theory, it is, of necessity, a relativistic theory.* The equation of our theory does not just duplicate QM, it indicates an inherent nonlinearity in QM which is subject to experimental verification. We show, therefore, that conventional QM is a corollary of classical deterministic principles. We suggest this concept of nature conflicts with that prevalent in modern physics.

1. INTRODUCTION AND SUMMARY

The question of a conceptual foundation for quantum mechanics (QM), at a less radical level than that of an "alleged" break with classical mechanics and electrodynamics, has become the subject of increasing interest recently—roughly since 1965. The theories to date have been generalizations of Einstein's analysis of Brownian motion to non-Markoffian stochastic processes whose job it is to impose a random vacuum interaction upon a "single" charged particle. We feel that these theories, besides being riddled with mathematical difficulties as in the various versions of stochastic electrodynamics, are too weak in that they cannot indicate the classical simplicity of QM.

No extraneous vacuum interaction underlies the theory to be developed. We do see, however, that the mathematics demands relativistic kinematics. In fact, the very physics that was considered insufficient to explain QM suffices to make of QM a mere theorem.

We will see that QM follows from quite economical principles in the form of equations (2.1) and (2.2) (below) alone such that (2.7), the

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Klein–Gordon equation, results. As (2.7) applies to a single classical particle "it is not QM" but yields QM as (2.11) on ensemble averaging. We will prove that a special case of (2.11) corresponds to conventional QM by expanding a solution of this (2.11) in terms of any complete orthonormal set of solutions of (2.7) such that conventional QM reveals itself to be of classical origin. So we have a nonlinear classical statistical QM in (2.11) as a consequence of (2.7), which in turn has the necessary form only within relativistic kinematics.

The theory of QM to be developed is based conceptually, in part, on an ensemble of mutually noninteracting classical particles. The typical member particle P^l must, however, be described relativistically; a Minkowski frame M^l is assigned to P^l alone. We distribute an electromagnetic field in each M^l that is represented by a P^l-unique Maxwell's 4-potential $\mathbf{a}^l$. We assume initially that there is no correlation of $\mathbf{a}^j$ and $\mathbf{a}^k$ for $j \neq k$. We will suppose, moreover, that all these P^l have identical rest mass, m, and electric charge, e, such that, in essence, one deals with exact particle copies; each in a distinct electromagnetic environment.

2. THE CLASSICAL MECHANICS OF AN ENSEMBLE

It is convenient at first to imagine an infinite number of Minkowski frames M^l with the same space-time origin. In an M^l, there is the world line of a single particle P^l whose proper time at 4-position $\mathbf{x}^l$ is s^l. If all the M^l are also allowed at rest with respect to an inertial frame $\overline{M}$ chosen at will, we refer to the collection of P^l as an ensemble. An ensemble member P^l can now be assumed transplanted from M^l to $\overline{M}$. As such we require that its world line be independent of all the other P^k present. At $\mathbf{x}^l(s^l)$ in $\overline{M}$, then, P^l mirrors its exact world line in M^l. By this detailed recipe we will be able to give rigor to an ensemble averaging that is central to our classical statistical theory of QM.

At all $\mathbf{x}$ of $\overline{M}$ in the world line of each P^l, the 4-velocity $\mathbf{v}^l$ has constant magnitude:

$$(\mathbf{v}^l)^2 = -1 \tag{2.1}$$

By use of Hamilton's principal function (action) ω^l, we relate $\mathbf{v}^l$, 4-momentum, and the specific Maxwell's 4-potential $\mathbf{a}^l$. We emphasize a distinct distribution of $\mathbf{a}^l$ in $\overline{M}$ for each P^l. The Lorentz force on P^l arises only from $\mathbf{a}^l$, same label, independent of other $\mathbf{a}^k$ distributions transplanted to $\overline{M}$:

$$mc\mathbf{v}^l = \Box\omega^l - \frac{e}{c}\mathbf{a}^l \tag{2.2}$$

An *exclusively relativistic* property of $\mathbf{v}^l$, in that its 4-divergence vanishes, is demonstrated by a Lemma as (2.14)→(2.19):

$$\square\cdot\mathbf{v}^l=0 \tag{2.3}$$

It must be understood that (2.3) is possible if and only if relativistic kinematics is used; *QM is relativistic*!!

Let us now consider some simple mathematics. By the use of an inverse relationship between $\log(\vartheta)$ and $\exp(\vartheta)$ one gives an alternate form to ω^l:

$$\omega^l=\frac{\hbar}{i}\log(\psi^l) \tag{2.4}$$

$$\psi^l=\exp\left(\frac{i}{\hbar}\omega^l\right) \tag{2.5}$$

The Hamilton–Jacobi equation for a P^l follows from (2.1) and (2.2) but if (2.3) is invoked it is equivalent to a Klein–Gordon equation for each P^l:

$$\begin{aligned}\left(\square\omega^l-\frac{e}{c}\mathbf{a}^l\right)^2 &= -\hbar^2\left(\frac{\square\psi^l}{\psi^l}\right)^2-2\frac{\hbar e}{ic}\mathbf{a}^l\cdot\frac{\square\psi^l}{\psi^l}+\frac{e^2}{c^2}(\mathbf{a}^l)^2\\ &\quad+\frac{\hbar}{i}\square\cdot\left(\square\omega^l-\frac{e}{c}\mathbf{a}^l\right)\\ &= -\hbar^2\left(\frac{\square\psi^l}{\psi^l}\right)^2-2\frac{\hbar e}{ic}\mathbf{a}^l\cdot\frac{\square\psi^l}{\psi^l}+\frac{e^2}{c^2}(\mathbf{a}^l)^2\\ &\quad-\hbar^2\left[\frac{\square^2\psi^l}{\psi^l}-\left(\frac{\square\psi^l}{\psi^l}\right)^2\right]-\frac{\hbar e}{ic}\square\cdot\mathbf{a}^l\\ &= \frac{1}{\psi^l}\left\{-\hbar^2\square^2\psi^l+\frac{e^2}{c^2}(\mathbf{a}^l)^2\psi^l\right.\\ &\quad\left.-\frac{\hbar e}{ic}\left[\mathbf{a}^l\cdot\square\psi^l+\square\cdot(\mathbf{a}^l\psi^l)\right]\right\}\end{aligned} \tag{2.6}$$

$$\left(\frac{\hbar}{i}\square-\frac{e}{c}\mathbf{a}^l\right)^2\psi^l+m^2c^2\psi^l=0 \tag{2.7}$$

It is clear how the surviving terms of (2.6) factored into the Klein–Gordon operator acting on ψ^{l}. If instead we first take an ensemble average of (2.6) $\circ\psi^{l}$, no strict factoring is possible. By so doing, though, one winds up with a nonlinear Klein–Gordon equation for the $\bar{\psi}$ distribution in $\bar{M}$. As such, we have $\overline{(2.6)\circ\psi}$ as (2.11) by use of these identities:

$$\overline{\mathbf{a}^2\psi} = (\bar{\mathbf{a}})^2\bar{\psi} + \mathrm{var}(\mathbf{a})\bar{\psi} + \mathrm{cov}(\mathbf{a}^2, \psi) \tag{2.8}$$

$$\overline{\mathbf{a}\cdot\Box\psi} = \bar{\mathbf{a}}\cdot\Box\bar{\psi} + \mathrm{cov}(\mathbf{a}\cdot\Box, \psi) \tag{2.9}$$

$$\Box\cdot(\overline{\mathbf{a}\psi}) = \Box\cdot(\bar{\mathbf{a}}\bar{\psi}) + \mathrm{cov}(\mathbf{a}\cdot\Box, \psi) - \underbrace{\mathrm{cov}(\psi, \Box\cdot\mathbf{a})}_{=0} \tag{2.10}$$

We have thus isolated all terms of $\bar{\psi}$ and $\bar{\mathbf{a}}$ in (2.6) so that the ensemble average of (2.7) becomes:

$$\left(\frac{\hbar}{i}\Box - \frac{e}{c}\bar{\mathbf{a}}\right)^2\bar{\psi} + m^2c^2\left[1 + \frac{e^2}{E^2}\mathrm{var}(\mathbf{a})\right]\bar{\psi} + Q(\mathbf{a}, \psi) = 0 \tag{2.11}$$

An important conceptual interpretation of $\mathbf{a}^{l}$ follows. We assume the $\mathbf{a}^{l}$ in (2.2) & (2.7) gives an exact photon interaction with each P^{l} at the latter's $\mathbf{x}$. The ensemble average, $\bar{\mathbf{a}}$, of (2.11) satisfies Maxwell's "already" statistical equations.

As as ensemble equation, (2.11) identifies E with a single particle's rest energy and Q with a nonlinear correction to the conventional QM formalism:

$$E = mc^2 \tag{2.12}$$

$$Q(\mathbf{a}, \psi) = \left(\frac{e}{c}\right)^2 \sum_{j=2}^{\infty} f_j(\overline{\mathbf{a}^2})\bar{\psi}^j - 2\frac{\hbar e}{ic}\sum_{k=2}^{\infty} g_k(\bar{\mathbf{a}}\cdot\Box)\bar{\psi}^k \tag{2.13}$$

The failure of ψ^{l} to represent a QM wave function in (2.7) is clear. The same is not so for $\bar{\psi}$ in (2.11) due to ensemble richnesses. So, contrary to conventional QM, where $\bar{\psi}$ and a linearized (2.11) are primitive concepts, we now suggest that QM is contained in mere classical statistical mechanics and electrodynamics; *their concepts, not* $\bar{\psi}$, *are primitive*!

A Lemma

It is our purpose to demonstrate that $\Box \cdot \mathbf{v}' = 0$, which as such follows from the equation of motion, written in terms of the Hamiltonian. If we write $\Box \omega = \mathbf{p}$ and define $\Box'$ as the 4-differential operator *wrt* $\mathbf{v}$, then our argument can be arranged successfully in (2.14)→(2.19).

In what follows, $\mathbf{I}$ is the unit 4-dyadic and $\mathbf{F}$ is the electromagnetic field 4-dyadic:

$$H = \frac{1}{2} mc\mathbf{v} \cdot \mathbf{v} \tag{2.14}$$

$$\mathbf{p} = mc\mathbf{v} + \frac{e}{c}\mathbf{a} \tag{2.15}$$

$$mc\mathbf{v} = mc\mathbf{v} \cdot \mathbf{I} = mc\mathbf{v} \cdot \Box' \mathbf{v} = \Box' H \tag{2.16}$$

$$\frac{d\mathbf{p}}{ds} = -\Box H = mc\frac{d\mathbf{v}}{ds} + \frac{e}{c}\frac{d\mathbf{a}}{ds} \tag{2.17}$$

$$\frac{d\mathbf{v}}{ds} = \frac{e}{mc^2}\mathbf{F} \cdot \mathbf{v} \tag{2.18}$$

$$\begin{aligned} mc\Box \cdot \mathbf{v} &= \Box \cdot \Box' H = \Box' \cdot \Box H = -\Box' \cdot \frac{d\mathbf{p}}{ds} \\ &= -mc\Box' \cdot \frac{d\mathbf{v}}{ds} - \frac{e}{c}\Box' \cdot \frac{d\mathbf{a}}{ds} \\ &= -\frac{e}{c}\Box' \cdot \mathbf{F} \cdot \mathbf{v} - \frac{e}{c}\Box' \cdot \frac{d\mathbf{a}}{ds} = 0 \end{aligned} \tag{2.19}$$

On the furthermost right-hand side of (2.19) the first term vanishes due to the skew nature of $\mathbf{F}$, while the second term vanishes because $\mathbf{a}$ is a function of $\mathbf{x}$.

As such, our argument depends explicitly on the electromagnetic interaction.

The analog of (2.19) obtains for 3-velocity in a nonrelativistic electromagnetic environment. We note, however, that Maxwell's equations and Newton's mechanics are not rigorously compatible in view of their distinct transformation invariance. So, the above analysis is *correct-in-principle* only for 4-velocity; in fact, we claim (2.3) is kinematic!!

3. ON A CLASSICAL STATISTICAL QM

The constraint var(**a**)$\neq 0 = Q(\mathbf{a}, \bar{\psi})$ gives structure to (2.11) of a "wave" equation for $\bar{\psi}$. We must also note that (2.7) is a "wave" equation for the ψ^l of each P^l. So, mere resemblance does not in itself grant a classical statistical reproduction of QM to the linearized (2.11) since (2.7) is not a QM equation.

We now see how a classical ensemble of particles may be constructed by decomposing a normalized $\bar{\psi}$ solution of the scatterless (2.11) in terms of some complete orthonormal set of functions $\varphi_n{}^{\dagger} = \alpha_n \varphi_n$. The φ_n are single particle solutions of (2.7). The relation (2.5) shows that φ_n cannot be normalized. The α_n assure that $\int (\varphi_j{}^{\dagger})^* \varphi_k{}^{\dagger} = \delta_{jk}$; discussed below. As such, it outlines a *heuristic* proof that the scatterless (2.11) is conventional QM; a statistic theory of classical origin. We assign to each independent solution, φ_n, of (2.7) an associated field of trajectories over all $\bar{M}$ by use of (2.4):

$$\bar{\psi} = \sum c_n \varphi_n{}^{\dagger} \tag{3.1}$$

$$mc\mathbf{v}_n = \frac{\hbar}{i} \Box \log(\varphi_n) - \frac{e}{c}\mathbf{a} \tag{3.2}$$

The subensemble fraction of particles with 4-velocities $\mathbf{v}_n$ is $|c_n|^2 = \int \rho_n$. The space time densities, ρ_n, must also satisfy (3.3), (3.4), and (3.5) with $\Box \cdot \mathbf{v}_n = 0$:

$$mc\bar{\mathbf{v}} + \frac{e}{c}\mathbf{a} = \frac{\hbar}{2i} \Box \log(\bar{\psi}/\bar{\psi}^*) = \frac{\hbar}{i}\left(\sum \rho_n\right)^{-1} \sum \rho_n \Box \log(\varphi_n) \qquad (3.3)^1$$

$$0 = \Box \cdot (\rho_n \mathbf{v}_n) \tag{3.4}$$

$$\bar{\psi}\bar{\psi}^* = \sum \rho_n = \rho \qquad (3.5)^2$$

It is evident that, given a set φ_n, the countable infinite number of simultaneous equations derived from all $|c_n|^2$, as well as (3.4), are "satisfied" by variable $\rho_n(\mathbf{x})$.

[1]The right-hand side is dictated by considering the limit of one particle.

[2]The left-hand side is justified in that 4-current $\rho\bar{\mathbf{v}}$ is to be conserved.

We seek φ_n which solves (3.6), the Hamilton–Jacobi equation for $\log(\varphi_n)$, and (2.7), the Klein–Gordon equation obeyed by a single classical particle. The only property required of each $\log(\varphi_n)$ is that it be purely imaginary. The way we obtain a complete independent set of solutions, φ_n, of (2.7) is by first solving (3.6) for $-i\hbar\log(\varphi_n)$, or action, with distinct boundary conditions on an initial spacelike surface. By so doing, the $\log(\varphi_n)$ are almost sure of being independent *wrt* **x** in $\overline{M}$.

$$\left[\frac{\hbar}{i}\Box\log(\varphi_n)-\frac{e}{c}\mathbf{a}\right]^2+m^2c^2=0=\Box^2\log(\varphi_n) \tag{3.6}$$

By judicial juggling "real" α_n, all $\varphi_n{}^{\dagger}=\alpha_n\varphi_n$ can be orthonormal, with α_n and ρ_n loosely related via $|c_n|^2=\int\rho_n$.

We point out that for a chosen independent set of φ_n, the α_n are not unique. We also note that once a set of α_n is decided, the ρ_n are not unique either. The decision of α_n is not discussed further.

We conjecture that φ_n in (3.6) may be found such that all ρ_n are nonnegative. We suggest φ_n which only allow some negative ρ_n be ruled out as acceptable. We believe there always exist sets $\{\varphi_n\}$ and $\{\alpha_n\}$ such as to allow $\rho_n=|c_n|^2(\alpha_n)^2$.

As such, though what we have just outlined does not constitute a rigorous proof, it indicates *QM is classical*!

We next look at the expression $\mathbf{j}=\rho\bar{\mathbf{v}}$. The 4-vector $\bar{\mathbf{v}}$ is not conserved, yet $\bar{\mathbf{v}}$ is unique in form, as demanded by (3.3). So, we assign to ρ the form (3.5) since $\Box\cdot\mathbf{j}=0$:

$$\mathbf{j}=(mc)^{-1}\left[\frac{\hbar}{2i}(\bar{\psi}^*\Box\bar{\psi}-\bar{\psi}\Box\bar{\psi}^*)-\frac{e}{c}\bar{\psi}\bar{\psi}^*\mathbf{a}\right] \tag{3.7}$$

By so doing, we have introduced an alternative consistent theory of 4-current for the linearized (2.11); a Klein–Gordon equation. Or, specifically, $\rho=\bar{\psi}\bar{\psi}^*$ is ensemble density at **x**. We note (3.7) is a consequence of the linear (2.11). It is not contained in the full nonlinear classical QM.

Let us consider one more implication on the thesis that QM is contained in the linear limit of (2.11) as follows. At no point have we been required to modify Maxwell's 4-potential on each P^l. In fact, for $\text{var}(\mathbf{a})=0=Q(\mathbf{a},\psi)$, there is no difference in the electromagnetic environment of any P^l at a given **x** in $\overline{M}$. As such, it appears that the only randomness consistent with our classical statistical theory of QM is a choice of $\mathbf{v}^l$ and s^l for a typical P^l at $\mathbf{x}$.

It is clear that our classical statistical QM is not a stochastic theory. In fact, it is stronger than theories of stochastic electrodynamics inasmuch as these latter do generalize classical physics to allow for a non-Markoffian process attributed to a zero point "vacuum." No stochastic "vacuum" attends our treatment. In this sense, we managed to synthesize QM from just that physics already available to its inventors. Of the available physics, the need for relativistic kinematics is most remarkable. On this note then, QM is a mere corollary to a straightforward theorem in a statistical and relativistic Hamilton–Jacobi theory; *electrodynamics plays no active role*!

4. CONCLUSION

We have seen how a "wave" equation for a single particle, (2.7), results from Hamilton–Jacobi theory only on invoking the kinematics of special relativity. In brief, our (2.18) leads to vanishing of the 4-divergence of $\mathbf{v}^l$ as (2.3) subject to ω^l as in (2.2), such that substitution of (2.4) and (2.5) in (2.2), and (2.2) in (2.1), with (2.3), yields (2.6), then (2.7) by factoring. It may be that the existence of $\mathbf{a}^l$ has no bearing on the fact that (2.7) resembles a "wave" equation. The existence of $\mathbf{a}^l$, distinct and unique to a P^l, leads us to an ensemble-averaged nonlinear Klein–Gordon equation (2.11), as a consequence of (2.7). As we suggested, any $\bar{\psi}$ of the linearized (2.11) can be put in correspondence with a classical ensemble, unique only to the decomposition chosen for $\bar{\psi}$; this gave us a heuristic proof that *QM is of classical statistical nature*.

The full nonlinear QM, (2.11), was not discussed as it will be the subject of a future work. We can, however, make an observation on a linear limit of (2.11) which is a generalized Klein–Gordon equation. If $\mathrm{var}(\mathbf{a})\neq 0$ but $Q(\mathbf{a}, \psi)=0$, then the mass term is affected. It is a viable constraint. We thus see that $\mathrm{var}(\mathbf{a})\neq 0$, which in itself does not contribute to the quantum mechanism, is subject to experimental test as $\mathrm{var}(\mathbf{a})$ depends on the 4-potential used.

We interpret the apparent quantization of states in terms of a simple classical picture. The world lines of individual elementary particles are capable of complete descriptions in microphysics. The statistical equation (2.11), indicates that an eigenstate is never achieved exactly by a single P^l for nonzero elapse of its s^l. So, the nonrelativistic limit of the linearized (2.11) suggests that energy eigenstates are mathematical idealizations. It seems to us that eigenstates of energy are highly "preferred" stable configurations under a specified electromagnetic potential as $s^l \to \infty$. The gross behavior of classical ensembles cannot be overestimated!

The usual way of speaking about nonrelativistic states of a hydrogen atom refers to a QM electron as being spread out in probability of being found. We suggest instead that the familiar probability pertains to the 3-density of an ensemble of classical electrons in statistical "equilibrium". The s state, for example, is interpreted to have vanishing values of ensemble averaged 3-velocity and angular momentum though individual members are in strict determined paths. It should be clear, however, that an s state is achieved as a limit for infinite elapsed time. The emission of a photon from a p to an s state appears as an indication of a "statistical" activity. By a "statistical" activity we see some electrons subjected to a short and sharp deviation from a strict Coulomb field. If these few rapid transitions are considered apart from the rest of the ensemble, we understand "line" spectra. We need only assign to Maxwell's equations an "already" statistical quality for the existence of photons.

5. APPENDIX

So far we have neglected a precise definition of ensemble averaging. It will prove convenient for us to associate with averaging, the number of worldlines N inside an infinitesimal volume V about **x**. Let V be a dimensionless quantity:

$$\bar{\psi} \cong V^{-\frac{1}{2}} \sum_{1}^{N} \psi^{l} = V^{-\frac{1}{2}} \sum_{1}^{N} \exp\left(\frac{i}{\hbar}\omega^{l}\right) \tag{5.1}$$

$$\bar{\psi}^{*} \cong V^{-\frac{1}{2}} \sum_{1}^{N} \exp\left(-\frac{i}{\hbar}\omega^{l}\right) \tag{5.2}$$

$$\bar{\psi}\bar{\psi}^{*} \cong V^{-1}\left\{N + 2\sum_{j=1}^{N}\sum_{k>j}^{N} \cos\left[\frac{1}{\hbar}(\omega^{j}-\omega^{k})\right]\right\} \rightarrow \rho \tag{5.3}$$

The above are viable definitions!

In (5.3), N and V are constrained in such a way that cosine terms "cancel" yet the ratio remains finite. In this way, by our specific convention for averaging, we identify the left-hand side of (5.3) with ensemble density ρ at **x**. The required property for (3.5) is thus recovered independent of conventional QM. It was seen that this relation be consistent, while agreeing with experience. If (5.3) is multiplied into (3.3), the result can be shown to satisfy particle conservation:

$$\rho\bar{\mathbf{v}}=(mc)^{-1}\left[\frac{\hbar}{2i}(\bar{\psi}^{*}\Box\bar{\psi}-\bar{\psi}\Box\bar{\psi}^{*})-\frac{e}{c}\bar{\psi}\bar{\psi}^{*}\mathbf{a}\right] \tag{5.4}$$

We note that, though self consistent, our form for ρ is at odds with that usually attached to the Klein–Gordon equation. It is quite superior in that, unlike the usual form, it is always unambiguously nonnegative. As such, it constitutes an alternative theory of the scatterless (2.11).

We will explore the connection between (5.1) and Feynman's sum-over-paths approach to QM in what follows.

The action, ω^{l}, of a P^{l} whose worldline contains $\mathbf{x}_1$, $\mathbf{x}_2$, and $\mathbf{x}_3$ in $\overline{M}$ obeys an additive rule:

$$\omega^{l}(3,1)=\omega^{l}(3,2)+\omega^{l}(2,1) \tag{5.5}$$

The associated ψ^{l} obeys a multiplicative rule by (2.4) and (2.5):

$$\psi^{l}(3,1)=\psi^{l}(3,2)\psi^{l}(2,1) \tag{5.6}$$

We have an ensemble average for all P^{l} whose worldlines go through $\mathbf{x}_1$ and $\mathbf{x}_3$ as sums over all independent $\mathbf{x}_2$. By this, we mean that only one $\mathbf{x}_2$ is chosen for each P^{l}, such as those lying on a spacelike surface separating $\mathbf{x}_1$ and $\mathbf{x}_3$:

$$\bar{\psi}(3,1)=\operatorname{Lim} V_1^{-\frac{1}{4}}V_3^{-\frac{1}{4}}\sum_{\mathbf{x}_2}\psi^{l}(3,2)\psi^{l}(2,1) \tag{5.7}$$

In the above, V_1 and V_3 are infinitesimal volumes about $\mathbf{x}_1$ and $\mathbf{x}_3$, respectively, containing worldlines that start in V_1 and end in V_3.

We conjecture that the right-hand side of (5.7) is nothing less than a relativistic path integral. So the equal weight assigned to each P^{l} suggests the form of our (5.3); $\rho=\bar{\psi}\bar{\psi}^{*}$.

The heuristics, represented by (5.1)→(5.4) and (5.5)→(5.7), respectively, are outside the scope of our development leading to (2.7) and (2.11).